We examine the construction and application of projectively transformed diagrams of state for binary mixtures in the case of a constant total pressure. A new form of the enthalpy diagram - the composition of the humid air - is presented in Appendix 1.

Introduction. In this paper we examine a rational method known in England as the Stroud Convention [ 1,2$]$ and in Germany as Rechnen mit Grössengleichungen [3], on the basis of which we introduce the product of certain units of measurement in such a manner that the quantities $G_{A}$ and $G_{B}$ of two substances $A$ and $B$ are assumed to exhibit different physical dimensions. As a consequence of this concept we find that -the quantities $G_{A}$ and $G_{B}$ cannot be compared, and moreover it becomes illogical to use such quantities as, for example, "a kg of mixture," since the relative composition of the mixture changes during the course of the examination.

Projective Transformations of Scales for Binary-Mixture Diagrams. We examine a diagram (see Fig. 1) in which the coordinates represent the quantities $G_{A}$ and $G_{B}$ forming the mixture, with $G_{A}$ and $G_{B}$ expressed in units denoted by $U_{A}$ and $U_{B}$. In specific cases $U_{A}$ and $U_{B}$ must be replaced by the corresponding notation which indicates the dimensions of the units and the identification of the substances, e.g., kg of dry air or kg (or g ) of $\mathrm{H}_{2} \mathrm{O}$. If the quantities A and B are understood to refer to flows, the quantities $\mathrm{G}_{\mathrm{A}}$ and $\mathrm{G}_{\mathrm{B}}$ can represent the quantities $A$ and $B$ passing through some transverse cross section with a certain interval of time. Moreover, we can introduce new notation, let us say $\dot{G}_{A}$ and $\dot{G}_{B}$, and analogously $\dot{U}_{A}$ and $\dot{U}_{B}$.

The points on the positive coordinate axes in the figure represent, respectively, the quantities $A$ and $B$. All of the points for any straight ray from the coordinate origin to the quadrant between these two halfaxes at the coordinates represent a mixture of the same relative composition, with the relationship between the coordinates for each point on the ray remaining the same.

The various relative compositions can thus be described by a cluster of rays emanating from the coordinate origin, as shown in the figure. However, since each ray is uniquely defined by any of its points in space, another more practical method of describing the sequence for the relative composition is doubtlessly represented by a series of such points, which correspond to the intersection of the rays by any chosen straight line drawn through these points. The lines intersecting the cluster of rays will subsequently be referred to as the "scale lines." As an example the figure shows four scale lines, beginning with the point $\mathrm{G}_{\mathrm{A}}=1 \mathrm{U}_{\mathrm{A}}$ on the $\mathrm{G}_{\mathrm{A}}$ axis. Three of these lines are drawn to the point $\mathrm{G}_{\mathrm{B}}=\mathrm{k}$ on the $\mathrm{G}_{\mathrm{B}}$ axis, with k set, respectively, equal to $0.5 \mathrm{U}_{\mathrm{B}}, 1 \mathrm{U}_{\mathrm{B}}$, and $1.5 \mathrm{U}_{\mathrm{B}}$. For the sake of brevity, these lines may be referred to as the scale lines for $\mathrm{k}=0.5 \mathrm{U}_{\mathrm{B}}, 1 \mathrm{U}_{\mathrm{B}}$, and $1.5 \mathrm{U}_{\mathrm{B}}$. The fourth scale line is drawn parallel to the $\mathrm{G}_{\mathrm{B}}$ axis and it can be referred to as the scale line for $k=\infty$.

Construction and Calculation of Projectively Transformed Scales for a Component Ratio $\mathrm{x}=\mathrm{G}_{\mathrm{B}} / \mathrm{G}_{\mathrm{A}}$. The quantity $\mathrm{G}_{\mathrm{B}}$ on the scale line for $\mathrm{k}=\infty$ (see Fig. 1) varies linearly, whereas the quantity $\mathrm{G}_{\mathrm{A}}$ is constant. It is therefore logical to plot a uniform scale on this line for the variable x , expressing the relative composition by the relationship

$$
\begin{equation*}
x=G_{\mathrm{B}} / G_{\mathrm{A}} \tag{1}
\end{equation*}
$$

In this case the scale differs from the scale for $G_{B}$ on the axis of abscissas (see the figure) only in terms of the unit of measurement, and namely, instead of $U_{B}$ we have $U_{B} / U_{A}$.

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Fig. 1. Construction of the projectively transformed x scales.

The use of the variable x determined in this manner is particularly useful in processes with varying $x$, when some quantity $B$ is fed into or removed from the mixture, and where $G_{A}$ remains constant. As examples of such processes we can cite (in the cases of mixtures of dry air and $\mathrm{H}_{2} \mathrm{O}$ ) the addition of moisture to and the removal of moisture from humid air. The latter may be associated, for example, with the drying of a material, and also with the processes taking place in cooling towers, and the like.

Extending the rays, as shown in the figure, from the coordinate origin to the point corresponding to the sequence of $x$ values (on the $x$ scale) for $k=\infty$, we obtain a cluster of rays which can be enumerated, as can their end points on a uniform x scale, or its extension.

The sequence of $x$-enumerated points, formed by the intersection of the $x$-enumerated rays of the scale line drawn for the finite parameter $k$, for example, of one of the three indicated in the figure, obviously produces a projectively transformed nonuniform x scale.

To compare the nonuniform scales derived in this manner, we can examine the distances between the scale points from the lines for $\mathrm{k}=\infty$ in the form

$$
\begin{equation*}
y=1-G_{\mathrm{A}} / U_{\mathrm{A}}, \tag{2}
\end{equation*}
$$

where $G_{A}$ represents the ordinate of the point scale under consideration, while $1 U_{A}$ represents the maximum value for the corresponding ordinate. We see from the figure that the smaller the parameter k of the projectively transformed $x$ scale, the more stretched is the scale in the region of small numbers $x$ at the cost of compression of the scale in the region of large $x$ values. On the other hand, the greater the parameter k , the larger the portion of the scale that assumes a uniform nature.

Formulas for the Calculation of Projectively Transformed x-Scales. The diagram in the figure is intended primarily for a visual representation of the principle involved in the construction of projectively transformed $x$ scales. The precise construction of such a scale, with the chosen parameter $k$, is a rather cumbersome operation. A more practical approach is the calculation - using formula (2) - of the coordinate points of the scale from its origin ( $x=0$ ), using the total length of the scale as the unit. We then have to lay out the positions of the coordinate points, using a ruler and a compass.

The formula required for the calculation of $y$ as a function of $x$ for the chosen parameter $k$ is easily derived in the following manner. The equation for the scale line determined by the parameter k , as shown in the figure, is the following:

$$
G_{\mathrm{A}} / U_{\mathrm{A}}+G_{\mathrm{B}} / k=1
$$

It is linear with respect to the variables $G_{A}$ and $G_{B}$ and yields $G_{A}=1 U_{A}$, if $G_{B}=0$, and $G_{A}=0$, if $G_{B}=k$, which is what we required. Replacing $G_{B}$ by $G_{A} x$ in this equation in accordance with (1), we derive the equation $\left(G_{A} / U_{A}\right)\left(1+U_{A} x / k\right)=1$ and hence $G_{A} / U_{A}=1 /\left(1+U_{A} x / k\right)$. Substituting this expression in the place of $\mathrm{G}_{\mathrm{A}} / \mathrm{U}_{\mathrm{A}}$ in (2), we obtain the sought formula which can be written in the form

$$
\begin{equation*}
y=\frac{U_{\mathrm{A}} x / k}{1+\bar{U}_{\mathrm{A}} x / k} \tag{3}
\end{equation*}
$$

For example, if $\mathrm{x}=0.3 \mathrm{U}_{\mathrm{B}} / \mathrm{U}_{\mathrm{A}}$ and $\mathrm{k}=0.5 \mathrm{U}_{\mathrm{B}}$, then $\mathrm{U}_{\mathrm{A}} \mathrm{x} / \mathrm{k}=\left(\mathrm{U}_{\mathrm{A}} \cdot 0.3 \mathrm{U}_{\mathrm{B}} / \mathrm{U}_{\mathrm{A}}\right) / 0.5 \mathrm{U}_{\mathrm{B}}=0.3 / 0.5=0.6$ (we thus verify the dimensional accuracy of the formula). With consideration of (3) we obtain $y=0.6 / 1.6$ $=0.375$, which corresponds to the data in Fig. 1, but is more exact.

A simple relationship exists between the parameter $k$ and the quantity $x_{1 / 2}$ at the center point of the $x$ scale, viz., for $y=1 / 2$. This relationship can be found by solving (3) for $k$ after introducing $x=x_{1} / 2$ and $y=1 / 2$, and writing it in the form

$$
\begin{equation*}
k=U_{\mathrm{A}} x_{1 / 2} \tag{4}
\end{equation*}
$$

After introduction of this expression for $k$ into (3) we obtain a simpler formula, i.e.,

$$
\begin{equation*}
y=\frac{x / x_{1 / 2}}{1+x / x_{1 / 2}} \tag{5}
\end{equation*}
$$

$\mathrm{i}, \mathrm{x}$-Diagram and Their Projective Transformations. In a two-dimensional coordinate system that is not necessarily orthogonal and referred to in this case as the i,x-diagram, the lines of one ordinate - known as $x$ lines - may represent various relative compositions $x=G_{B} / G_{A}$ of a binary mixture of two components $A$ and $B$. The other ordinate - known as the $i$ line - may represent the values for the variable $i$, defined as the total enthalpy of the mixture per unit $U_{A}$ of its content $G_{A}$ of the substance $A$. The original magnitude of $G_{A}$ thus corresponds to the quantity in the expression $x=G_{B} / G_{A}$. The point $i$, corresponding to the coordinate origin, refers to the pure component $A$ in the case of a specified constant total pressure $P_{\text {tot }}$ and, for example, at $0^{\circ} \mathrm{C}$.

Diagrams of this type, in which an oblique-angled coordinate system is used are convenient in practice (they are neither too long nor narrow). Such a diagram was developed for the case of humid air approximately 50 years ago by Ramzin who presented it in the form of the familiar I-d-diagram in his lectures in 1918, publishing it in 1925-1927, independently of Mollier whose diagram was published in 1923 and $1929[5,6]$. This well-known type of diagram has been used for a long time in calculating the processes involved in the handling of humid air. It will be encountered in the following as the "i,x-diagram of the conventional type."

A drawback of these diagrams becomes apparent when a uniform scale is used here for the variable x . Such a scale is not always practical. It may represent large values of x with unnecessarily great relative accuracy, whereas, on the other hand, the same scale in the region of low values for x may be somewhat too dense to permit calculation with the desired relative accuracy.

However, as follows from the previous, this drawback can be theoretically eliminated or, at the very least, substantially reduced in practice by choosing a corresponding projectively transformed $x$-scale. It would be interesting to examine how such a transformation affects the form of the entire $\mathbf{i}, \mathrm{x}$-diagram with its lines and curves for various values of the variables of interest to us.

Representation of Central Projections in Three-Dimensional Space. First of all, let us assume that a $G_{A}, G_{B}$-diagram as shown in the figure is contained in the horizontal plane referred to as the horizontal zero plane. The $i, x$-diagram usually intended for projective transformations is theoretically positioned in the vertical plane raised through the scale line for $k=\infty$ in the $G_{A}, G_{B}$ diagram so that the $x$ lines of this $i$, $x$-diagram are vertical. Each line must pass through the corresponding point on the uniform $x$ scale for $\mathrm{k}=\infty$. The indicated plane with its series of vertical equidistant lines for the corresponding values of x may be referred to as the plane for $\mathrm{k}=\infty$.

A nother plane, known as the k plane, can be raised vertically through the scale line for the correspondingly chosen value of $k$ in the horizontal $G_{A}, G_{B}$-diagram, i.e., for the value of $k$ which results in the nonuniform $x$ scale on the transformed $i, x$-diagram. The latter is determined on the selected $k$ plane by projecting the conventional type of $\mathrm{i}, \mathrm{x}$-diagram with the $\mathrm{k}=\infty$ plane onto the selected k plane, with the origin of the $G_{A}, G_{B}$-diagram serving as the center of the projection.

An example is presented in Appendix 1 of a projectively transformed $\mathbf{i}, \mathrm{x}$-diagram, achieved by carrying out the proposed operation in three-dimensional space.

The two variables $i$ and $x$ are of particular interest, since they have been chosen as independent variables of state and of the composition of the mixture. Let us find two families of straight lines which represent these two basic variables in the Appendix.

The Transformed $x$ Scale and the Associated Vertical x Lines. All of the vertical lines in the Appendix together make up a family of $x$ lines, numbered in accordance with the $x$ values noted at the bottom of the diagram.

The quantity $\mathrm{x}_{1 / 2}$ at the central point of the x scale, as we can demonstrate, is equal to $50 \mathrm{~g} \mathrm{H} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air. It thus follows from (4), where $U_{A}=1 \mathrm{~kg}$ of dry air, that the parameter k of the x scale is equal to $50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O}$. The x scale in the Appendix can be easily calculated with Eq. (5), with the unit 1 of $y$ corresponding to the total length of the scale.

Since it is the purpose of this projective transformation to reduce the shortcomings of the uniform $x$ scales on conventional types of $i, x$-diagrams, let us examine below the practical feasibility of a projectively transformed $x$ scale.

As we can see from the Appendix, the origin of the $x$ scale is virtually uniform and extended in comparison with its uniformly compressed extension. Beginning with x values of about $15 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air, the scale assumes an approximately logarithmic character, as is to be concluded from the approximately constant length of the intervals between the points of the scale for $\mathrm{x}=12.5,25,50,100$, and 200 $\mathrm{g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air, respectively, with the numbers forming a geometric progression. For the x values near $500 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air and higher we find pronounced compression of the scale, as a consequence of which the remaining portion of the scale is of little use for all intents and purposes. This shortcoming can be corrected, by constructing a diagram in analogous fashion, where the roles of the components dry air and $\mathrm{H}_{2} \mathrm{O}$ of the mixture are interchanged.

If the $\mathrm{x}_{1} / 2$ values used in the construction of the $\mathrm{i}, \mathrm{x}$-diagram are substantially lower than $50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air, the x scale must be drawn out even more at its origin because of the compression of its final segment, and vice versa. In special cases such modifications of the scale may prove to be useful. In calculations with humid air, in engineering practice, the x scale with $\mathrm{x}_{1 / 2}=50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ of dry air seems to be more convenient for extensive application.

Families of i Lines on Various Types of $i, x$-Diagrams. When a conventional oblique-angled i,xdiagram is placed in the vertical plane $\mathrm{k}=\infty$, the i lines are equidistant from the straight lines, with a steep slope to the right, providing a more satisfactory shape for the region of the diagram of interest to us.

In finding the central projection of the family of parallel lines from the vertical plane $\mathrm{k}=\infty$ onto the plane of the chosen finite $k$ we observe the following phenomenon. The central projection of the parallel lines in space onto a plane not parallel to these lines leads to projectively transformed straight lines (in that plane), and these all converge at a "common point." It is obvious that the i lines on the projectively transformed $i, x$-diagrams - beginning from the ordinates of the uniform $i$ scale on the vertical line $x=0$, must converge at some "common point" on the line for $x=\infty$. Any other position of the "common point" will obviously lead to absurd conclusions.

The steeper the $i$ line on a conventional oblique-angled $i, x$-diagram situated in the plane $k=\infty$, the lower the position of the "common point" of the i line on the corresponding projectively transformed diagram.

In the diagram given in the Appendix, the "common point" of the i line lies below the edge of the diagram, viz., at the zero points formed by the intersection of the (downward) extension of the uniform $\Delta \mathrm{i}$ $/ \Delta x$ scale and the vertical line for $x=\infty$.

Lines Transferred from a Conventional Type of i,x-Diagram to theProjectively Transformed Diagram. If the saturation curve ( $u=100 \%$ ) on the conventional type of $i, x$-diagram passes through an obliqueangled network of $i-$ and $x$-coordinate lines, the corresponding curve on the projectively transformed diagram can also be drawn through its projectively transformed network of $i$ and $x$ lines, with the first lines converging at a selected "common point," while the latter lines are vertical and nonequidistant.

However, since there is no final $x$-coordinate segment on the conventional type of $i, x$-diagram, the positions of certain auxiliary points on the curve under consideration should be calculated. The saturation curve ( $u=100 \%$ ) derived in this manner on the diagram in the Appendix is shown as a bold line.

If in the place of the curve we project a straight line, it turns out that the central projection of the straight line is a straight line, and in such cases, to fix the line, it is enough to have two points on the straight line.


Appendix 1. The $\mathrm{i}, \mathrm{x}$-diagram for humid air in the case of $\mathrm{P}_{\text {tot }}=1000 \mathrm{mbar}=750.06$ mm Hg . Tentative transformation of conventional-type diagrams proposed by Ramzin and Mollier. Calculated and constructed in 1952 by Westerberg under the the guidance of Professor Jarl Salin in the Department of Chemical Engineering at the Academy in Abo, Finland.

Isotherms above the Saturation Curve. The points above the saturation curve in the Appendix represent the state and compositions of the homogeneous gaseous mixtures of dry air and water vapor. If we treat these mixtures in the case of very low $\mathrm{P}_{\text {tot }}=1$ bar as mixtures of an ideal gas without dissociation, the isotherms are straight lines. In this case $i=h_{A}(t)+x h_{B}(t)$ is a linear function of $x$ for the given $t$, referred to the isotherm under consideration. The straight-line isotherms can thus be fixed by two points, e.g., for the dry air and the pure water vapor, and namely, for $x=0$ and $x=\infty$.

As we can see from the Appendix, the isotherms for low temperatures on the diagram rise to the right, but this rise is gradually slowed down at the higher temperatures and at around $70^{\circ} \mathrm{C}$ the ir slope reverses direction.

Curves of Constant "Degrees of Saturation" u. To illustrate the so-called relative humidities of the air, in the Appendix we present six thin lines for the values of $u=5,10,20,40,60$, and $80 \%$, with the "degree of saturation" defined in accordance with international recommendations [7] as the ratio $\mathrm{x} / \mathrm{x}_{\text {max }}$, where $x$ and $x_{\max }$ pertain to the same temperature $t$, with the latter corresponding to the case of saturation. The points on these curves are easily found without resort to diagrams of other types, in accordance with a method which can be described with a clear example. Thus, the choice of $t=52^{\circ} \mathrm{C}$ leads to convenient calculations, since for $50^{\circ} \mathrm{C}$ on the saturation curve we have $\mathrm{x}_{\max } \approx 100 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ da (dry air). The point on the isotherm for $t=52^{\circ} \mathrm{C}$ when $u=5,10,20, \ldots, 80 \%$, thus must correspond, respectively, to the points $\mathrm{x}=5,10,20, \ldots, 80 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg} \mathrm{da}$.

Determination of $\Delta \mathrm{i} / \Delta \mathrm{x}$ on the $\mathrm{i}, \mathrm{x}$-Diagram. When working with $\mathrm{i}, \mathrm{x}$-diagrams it is frequently desirable to find the values of $\Delta i / \Delta x=\left(i_{2}-i_{1}\right) /\left(x_{2}-x_{1}\right)$ in a simple manner, with the values of $i_{1}$ and $x_{1}$ representing the coordinates of some specified point $P_{1}$, while $i_{2}$ and $x_{2}$ are the coordinates of a second point $P_{2}$ on the diagram, or it might be desirable to find the solution of similar problems. To facilitate the solution of such problems, the i,x-diagrams of the type proposed by Mollier [5,6] are provided with a so-called "margin scale" on which the value of $\Delta i / \Delta x$ can be determined at the point at which the straight line cuts across the scale, with that line drawn from the origin of the $i, x$-diagram parallel to the straight line drawn through the points $P_{1}$ and $P_{2}$.

Use and Construction of the $\Delta \mathrm{i} / \Delta \mathrm{x}$ Scale Shown in the Appendix. The vertical lines for $\mathrm{x}=\infty$ in the tentatively transformed i,x-diagram appears partly within the limits of the diagram. This line for $\mathrm{x}=\infty$ may be, as demonstrated in the Appendix, provided with a $\Delta i / \Delta x$ scale, which frequently ensures the simpler solution of the $\Delta i / \Delta x$ problem than in cases in which a conventional type of $i, x$-diagram is used. For example, the problem of finding the value of $\Delta i / \Delta x=\left(i_{2}-i_{1}\right) /\left(x_{2}-x_{1}\right)$, which belongs to the pair of points $P_{1}$ and $P_{2}$ and thus to all pairs of points on the straight line through $P_{1}$ and $P_{2}$, can be solved simply by extending this line to the point at which it intersects the $\Delta i / \Delta x$ scale, on which we can read off the required value for $\Delta \mathrm{i} / \Delta \mathrm{x}$.

The use of the $\Delta i / \Delta x$ scale on the vertical line is based on the fact that the points of the scale can be regarded as points of flight with each of the points common to all of the straight lines which on the conventional diagram are parallel and characterized by the $\Delta \mathrm{i} / \Delta \mathrm{x}$ ratio under consideration.

The $\Delta i / \Delta x$ scale in the Appendix can be constructed in the following manner. The points at which the vertical line for $\mathrm{x}=\mathrm{x}_{1 / 2}=50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ da in the middle of the diagram, for example, intersects the lines for $i=15,16,17, \ldots \mathrm{kcal} / \mathrm{kg} \mathrm{da}$, as a consequence of the construction of the $i$ lines form a series of equidistant points on this vertical line. If we use the rays emanating from the origin in the case of $i=x=0$ to project this series of points onto the vertical line for $x=\infty$, a series of double points appears. The values of $\Delta \mathrm{i} / \Delta \mathrm{x}$ which relate to these points are fixed by the projection rays and can be easily calculated by using the values $\Delta \mathrm{x}=\mathrm{x}-0=50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ da and $\Delta \mathrm{i}=\mathrm{i}-0=15,16,17, \ldots \mathrm{kcal} / \mathrm{kg}$ da, belonging to the first halves of the projection rays. The end point of the rays on the line for $x=\infty$ can thus be numbered as follows:

$$
\frac{\Delta i}{\Delta x}=\frac{15,16,17, \ldots \mathrm{kcal} / \mathrm{kg} \mathrm{da}}{50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg} \mathrm{da}}=300,320,340 \mathrm{kcal} / \mathrm{kg} \mathrm{H}_{2} \mathrm{O}
$$

on the basis of the $\Delta i / \Delta x$ scale in the Appendix. Analysis shows that the scale is uniform, and that it can be extended in either direction.

Transformation of the Scale for $\Delta \mathrm{i} / \Delta \mathrm{x}$. The equation for the straight line on the tentatively trans formed $i, x$-diagrarn can be written in the form $i=i_{0}+b x$, where $i_{0}$ and $b$ are constants, with $b \neq 0$. This
line intersects the $\Delta i / \Delta x$ scale on the vertical line for $x=\infty$ at the point where

$$
\begin{equation*}
\frac{\Delta i}{\Delta x}=\frac{d i}{d x}=b=\frac{i-i_{0}}{x} . \tag{6}
\end{equation*}
$$

The variable $i$ is determined from the equation

$$
\begin{equation*}
i=\frac{G_{\mathrm{A}} h_{\mathrm{A}}+G_{\mathrm{B}} h_{\mathrm{B}}}{G_{\mathrm{A}}}=h_{\mathrm{A}}+x h_{\mathrm{B}} \tag{7}
\end{equation*}
$$

where $h_{A}$ and $h_{B}$ represent the specific enthalpy of pure $A$ and $B$. Replacing in in the right-hand member of (6) by the corresponding expression from (7) yields

$$
\begin{equation*}
\frac{\Delta i}{\Delta x}=\frac{h_{\mathrm{A}}+x h_{\mathrm{B}}-i_{0}}{x}=\frac{h_{\mathrm{A}}-i_{0}}{x}+h_{\mathrm{B}} \tag{8}
\end{equation*}
$$

If $\mathrm{x} \rightarrow \infty$ for the point moving along any selected line intersecting the vertical for $\mathrm{x}=\infty$ at the ordinate with a specified value of $\Delta \mathrm{i} / \Delta \mathrm{x}$, the first term in the right-hand member of (8) disappears and the equation assumes the form $\Delta i / \Delta x=h_{B}$. This indicates that the $\Delta i / \Delta x$ scale on the line for $x=\infty$ can also be presented as the scale for the specific enthalpy $h_{B}$ of the remaining pure component $B$, since the component $A$ is removed from the mixture. For the case of humid air, as cited in the Appendix, $h_{H_{2}} \mathrm{O}$ or $\Delta \mathrm{i} / \Delta \mathrm{x}$ are introduced as the variable scales on the line for $x=\infty$. In accordance with the definition of it is assumed that the zero point of $h_{H_{2}}$ o pertains to water at $0^{\circ} \mathrm{C}$ and $\mathrm{P}=\mathrm{P}_{\text {tot }}$.

Mixing Processes. If two binary mixtures A and B are mixed adiabatically when in the case of $\mathrm{P}_{\text {tot }}$, with one of the mixtures containing the components $G_{A_{1}}$ from $A$, and the other containing $G_{A_{2}}$ from the component $B$, the composition and states of the resulting mixture are represented by the single point $P_{m}$ on the conventional type of i,x-diagram. The last point, first of all, lies on the straight line connecting $P_{1}$ and $P_{2}$, representing the composition and state of the mixtures, and secondly, it divides this connecting line in inverse proportion to $\mathrm{G}_{\mathrm{A}_{1}}$ and $\mathrm{G}_{\mathrm{A}_{2}}$ according to the so-called "lever rule" [8]. If we use the transformed $\mathrm{i}, \mathrm{x}-$ diagram, the first statement is valid since straight lines have been transformed as straight lines; however, the second statement must be replaced by a recommendation for the calculation of the values of $\mathrm{x}_{\mathrm{m}}$ or $\mathrm{i}_{\mathrm{m}}$ on the basis of the "mixing rule":

$$
\begin{equation*}
x=\frac{x_{1} G_{\mathrm{A}_{2}}+x_{2} G_{\mathrm{A}_{2}}}{G_{\mathrm{A}_{1}}+G_{\mathrm{A}_{2}}} \tag{9}
\end{equation*}
$$

or

$$
\begin{equation*}
i_{m}=\frac{i_{1} G_{\mathrm{A}_{2}}+i_{2} G_{\mathrm{A}_{2}}}{G_{\mathrm{A}_{1}}+G_{\mathrm{A}_{2}}} \tag{10}
\end{equation*}
$$

and to find the point $P_{m}$ on the straight line from $P_{1}$ to $P_{2}$.
Mixtures of Water and Saturated Humid Air. The point $\mathbf{P}_{1}$ produced by the intersection of the isotherm $t$ and the saturation curve in the given $i, x$-diagram for $P_{\text {tot }}=1000$ mbar yields a maximum valuefor $x_{1}$ from the $x$ for a thermodynamically stable gaseous mixture of water vapor and dry air at a temperature $t$ and the given $P_{\text {tot }}$. An increase in the vapor content in the mixture for constant $t$ and $P_{\text {tot }}$ leads to a stable twophase mixture consisting of the gaseous phase of saturated humid air and the water phase, with the latter broken down into fine droplets reminiscent of fog.

The state of the liquid $\mathrm{H}_{2} \mathrm{O}$ phase in the two-phase mixture in the given $\mathrm{i}, \mathrm{x}$-diagram is represented by the point $\mathrm{P}_{2}$ on the downward extended uniform $\mathrm{h}_{\mathrm{H}_{2} \mathrm{O}}$ scale on the line for $\mathrm{x}=\infty$, where $\mathrm{h}_{\mathrm{H}_{2} \mathrm{O}}$ is equal to the specific enthalpy for the liquid $\mathrm{H}_{2} \mathrm{O}$ at a temperature.

The compositions and states of the two-phase mixtures achieved at a temperature of $t$ are shown in the $i, x$-diagram by the point on the straight line connecting $P_{1}$ and $P_{2}$. These lines, shown in the Appendix as dashed lines, are the isotherms of the temperatures under consideration in the "fog region," below the saturation line, and these may be referred to as "fog isotherms."

Five thin lines in the Appendix for $\mathrm{x}_{\mathrm{W}}=10,20,30,40$, and $50 \mathrm{~g} \mathrm{H}_{2} \mathrm{O} / \mathrm{kg}$ da are constructed to determine the quantity of water in the mixture per unit of dry air. The point on the curves for $\mathrm{x}_{\mathrm{W}}=$ const can be found for each of the selected fog isotherms, where $x$ is equal to the sum $x_{1}+x_{W}$, with $x_{1}$ pertaining to the value of $x$ at the point $P_{1}$ of the intersection between the fog isotherm and the saturation curve.

Approximate Wet-Bulb Thermometer Isotherms. The dashed fog isotherms in the Appendix are extended rectilinearly into the region above the saturation line, where they can be used as the wet-bulb thermometer isotherms in psychrometric investigations. It is assumed in accordance with Mollier that the socalled Lewis factor is equal to unity. To refine these hypotheses, we can turn, for example, to [9].

Related Diagrams. Spalding presented an i,x-diagram compiled by projective geometry and used in humid-air calculations [10]. The author of this article examined this diagram in comparison with that shown in the Appendix and made his comments in [11]. The author is indebted to his former assistant N. Westerberg for his thorough calculation and the completion (in 1952) of the original drawing of the diagram shown in the Appendix. A correspondingly large diagram with i values expressed in $\mathrm{kJ} / \mathrm{kg}$ da was calculated and plotted in 1957 by Lindell. Soininen is responsible for the conception, calculation, and plotting of the $\mathrm{i}, \mathrm{x}$ diagram in which the $i$ scale for $x=0$ has also been transformed and in which the x lines converge and in which we find nonuniform scales for $i$ and $\Delta i / \Delta x$; this diagram was published in a Finnish handbook [12]. VALMET in Abo (Finland) published a similar diagram. The author is indebted to his former as sistants Soininen and Lindell for their skillful work and assistance.

The principles and methods of graphical representation for the properties of two-phase mixtures consisting of humid air and water can doubtlessly be easily modified for cases in which ice is used in place of water. A modified i,x-diagram for humid air at a temperature of up to $-24^{\circ} \mathrm{C}$ has recently been published by EKONO [Association for Power and Fuel Economy] in the city of Helsingfors (Finland) in its program for 1968.

## NOTATION

$G_{A}$ and $G_{B}$
$U_{A}$ and $U_{B}$
$x$
$y$
$x_{1 / 2}$
$i$
$P_{\text {tot }}$
$t$
$u$
are, respectively, the quantities of the substances $A$ and $B$ in the binary mixture;
are, respectively, the chosen units for $G_{A}$ and $G_{B}$;
is the value (parameter) $G_{B}$ for the point at which the selected scale line intersects the $\mathrm{G}_{\mathrm{B}}$ axis (see Fig. 1);
is the $\mathrm{G}_{\mathrm{B}} / \mathrm{G}_{\mathrm{A}}$ ratio;
is the distance from the coordinate origin $x=0$ of the projectively transformed $x$ scale to the point $x$, proportional to the total length of the scale (Eq. (2));
is the x value at the center of the projectively transformed x scale, where $y=1 / 2$;
is the total enthalpy of the mixture per unit of its content $G_{A}$ of $A$;
is the total pressure of the mixture;
is the temperature of the mixture;
is the degree of saturation $x / x_{\max }$ for the gaseous mixture at the given temperature (the relative humidity of the air), where $x_{\max }$ corresponds to the state of saturation;
$\Delta \mathrm{i} / \Delta \mathrm{x}=\left(\mathrm{i}_{2}-\mathrm{i}_{1}\right) /\left(\mathrm{x}_{2}-\mathrm{x}_{1}\right)$
$\mathrm{h}_{\mathrm{A}}, \mathrm{h}_{\mathrm{B}}$, and $\mathrm{H}_{\mathrm{H}_{2} \mathrm{O}}$
$\mathrm{x}_{\mathrm{W}}$
are the symbols which pertain to the two points on the $\mathrm{i}, \mathrm{x}$-diagram;
are, respectively, the specific enthalpies of pure A and B, as well as of pure $\mathrm{H}_{2} \mathrm{O}$;
is the ratio of the water content in liquid form to the dry-air content in humid air (in the fog).

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